التكامل بالتجزيء

Integration by Parts

المجموعة A تمارين مقالية

في التمارين (1-14)، أوجد التكامل.

(1)

$$\int x \cos(3x)\, dx$$

$$u = x, \qquad\qquad dv = \cos(3x)\, dx$$

$$du = dx, \qquad\qquad v = \frac{1}{3}\sin(3x)$$

$$\int u\, dv = uv - \int v\, du$$

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$$\int x \cos(3x)\, dx = \frac{1}{3}x\sin(3x) - \int \frac{1}{3}\sin(3x)\, dx$$

$$= \frac{1}{3}x\sin(3x) + \frac{1}{9}\cos(3x) + C$$

(2)

$$\int x \sin(5x)\, dx$$

$$u = x, \qquad\qquad dv = \sin(5x)\, dx$$

$$du = dx, \qquad\qquad v = -\frac{1}{5}\cos(5x)$$

$$\int u\, dv = uv - \int v\, du$$

$$\int x \sin(5x)\,dx = -\frac{1}{5}x\cos(5x) + \int \frac{1}{5}\cos(5x)\,dx$$

$$= -\frac{1}{5}x\cos(5x) + \frac{1}{25}\sin(5x) + C$$

(3)

$$\int xe^{x-3}\,dx$$

$$u = x, \qquad\qquad dv = e^{x-3}dx$$

$$du = dx, \qquad\qquad v = e^{x-3}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int xe^{x-3}\,dx = xe^{x-3} - \int e^{x-3}\,dx$$

$$= xe^{x-3} - e^{x-3} + C = (x-1)e^{x-3} + C$$

(4)

$$\int (x-5)e^{x-5}\,dx$$

$$u = x - 5, \qquad\qquad dv = e^{x-5}dx$$

$$du = dx, \qquad\qquad v = e^{x-5}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int (x-5)e^{x-5}\,dx = (x-5)e^{x-5} - \int e^{x-5}\,dx$$

$$= (x-5)e^{x-5} - e^{x-5} + C = (x-6)e^{x-5} + C$$

(5)

$$\int \ln \sqrt[4]{x}\, dx$$

$$u = \ln \sqrt[4]{x} = \frac{1}{4} \ln x, \qquad\qquad dv = dx$$

$$du = \frac{1}{4x} dx, \qquad\qquad v = x$$

$$\int u\, dv = uv - \int v\, du$$

$$\int \ln \sqrt[4]{x}\, dx = x \ln \sqrt[4]{x} - \int \frac{1}{4} dx = x \ln \sqrt[4]{x} - \frac{1}{4} x + C$$

(6)

$$\int \ln(2x - 1)\, dx$$ www.kwedufiles.com

$$u = \ln(2x - 1), \qquad\qquad dv = dx$$

$$du = \frac{2}{2x - 1} dx, \qquad\qquad v = x$$

$$\int u\, dv = uv - \int v\, du$$

$$\int \ln(2x - 1)\, dx = x \ln(2x - 1) - \int \frac{2x}{2x - 1} dx$$

$$= x \ln(2x - 1) - \int \frac{2x - 1 + 1}{2x - 1} dx$$

$$= x \ln(2x - 1) - \int \left(1 + \frac{1}{2x - 1}\right) dx$$

$$= x \ln(2x - 1) - x - \frac{1}{2} \ln|2x - 1| + C$$

$$= x \ln(2x - 1) - x - \frac{1}{2} \ln(2x - 1) + C$$

(7)

$$\int (2x + 1)\ln(x + 1) \, dx$$

$$u = \ln(x + 1), \qquad\qquad dv = (2x + 1)dx$$

$$du = \frac{1}{x + 1} dx, \qquad\qquad v = x^2 + x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int (2x + 1)\ln(x + 1) \, dx = (x^2 + x)\ln(x + 1) - \int \frac{x^2 + x}{x + 1} dx$$

$$= (x^2 + x)\ln(x + 1) - \int \frac{x(x + 1)}{x + 1} dx$$

$$= (x^2 + x)\ln(x + 1) - \int x \, dx$$

$$= (x^2 + x)\ln(x + 1) - \frac{x^2}{2} + C$$

(8)

$$\int \frac{\ln x}{x^2} dx$$

$$u = \ln x, \qquad\qquad dv = \frac{1}{x^2} dx = x^{-2} dx$$

$$du = \frac{1}{x} dx, \qquad\qquad v = -\frac{1}{x}$$

$$\int u \, dv = uv - \int v \, du$$

$$\int \frac{\ln x}{x^2} dx = -\frac{1}{x}\ln x + \int x^{-2}dx = -\frac{1}{x}\ln x - \frac{1}{x} + C$$

(9)

$$\int \frac{\ln x}{\sqrt[3]{x}}dx$$

$$u = \ln x, \qquad\qquad dv = \frac{1}{\sqrt[3]{x}}dx = x^{-\frac{1}{3}}dx$$

$$du = \frac{1}{x}dx, \qquad\qquad v = \frac{3}{2}x^{\frac{2}{3}}$$

$$\int u\, dv = uv - \int v\, du$$

$$\int \frac{\ln x}{\sqrt[3]{x}}dx = \frac{3}{2}x^{\frac{2}{3}}\ln x - \int \frac{3}{2}x^{-\frac{1}{3}}dx = \frac{3}{2}x^{\frac{2}{3}}\ln x - \frac{9}{4}x^{\frac{2}{3}} + C$$

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(10)

$$\int x^2 \ln x^2 \, dx$$

$$u = \ln x^2 = 2\ln x, \qquad\qquad dv = x^2 dx$$

$$du = \frac{2}{x}dx, \qquad\qquad v = \frac{x^3}{3}$$

$$\int u\, dv = uv - \int v\, du$$

$$\int x^2 \ln x^2 \, dx = \frac{x^3}{3}\ln x^2 - \int \frac{2}{3}x^2 dx = \frac{x^3}{3}\ln x^2 - \frac{2}{9}x^3 + C$$

(11)

$$\int (x^2 - 2x) \cos x \, dx$$

$$u = x^2 - 2x, \qquad\qquad dv = \cos x \, dx$$

$$du = (2x - 2)dx, \qquad\qquad v = \sin x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int (x^2 - 2x) \cos x \, dx = (x^2 - 2x) \sin x - \int (2x - 2) \sin x \, dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int (2x - 2) \sin x \, dx$$

$$u = 2x - 2, \qquad\qquad dv = \sin x \, dx$$

$$du = 2dx, \qquad\qquad v = -\cos x$$

$$\int (2x - 2) \sin x \, dx = -(2x - 2) \cos x + \int 2 \cos x \, dx$$

$$= -(2x - 2) \cos x + 2 \sin x + C \qquad (2)$$

من **(1)** , **(2)** نحصل على:

$$\int (x^2 - 2x) \cos x \, dx = (x^2 - 2x) \sin x + (2x - 2) \cos x - 2 \sin x + C$$

$$= (x^2 - 2x - 2) \sin x + (2x - 2) \cos x + C$$

(12)

$$\int (x^2 + 3x) \sin x \, dx$$

$$u = x^2 + 3x, \qquad\qquad dv = \sin x \, dx$$

$$du = (2x + 3)dx, \qquad v = -\cos x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int (x^2 + 3x) \sin x \, dx = -(x^2 + 3x) \cos x + \int (2x + 3) \cos x \, dx \qquad (1)$$

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نستخدم القاعدة مرة ثانية لإيجاد:

</div>

$$\int (2x + 3) \cos x \, dx$$

$$u = 2x + 3, \qquad dv = \cos x \, dx$$

$$du = 2dx, \qquad v = \sin x$$

$$\int (2x + 3) \cos x \, dx = (2x + 3) \sin x - \int 2 \sin x \, dx$$

$$= (2x + 3) \sin x + 2 \cos x + C \qquad (2)$$

<div dir="rtl">

من (2) , (1) نحصل على:

</div>

$$\int (x^2 + 3x) \sin x \, dx = -(x^2 + 3x) \cos x + (2x + 3) \sin x + 2 \cos x + C$$

$$= (-x^2 - 3x + 2) \cos x + (2x + 3) \sin x + C$$

(13)

$$\int x^2 e^{x+1} \, dx$$

$$u = x^2, \qquad dv = e^{x+1} dx$$

$$du = 2x dx, \qquad v = e^{x+1}$$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^2 e^{x+1} \, dx = x^2 e^{x+1} - \int 2x e^{x+1} \, dx \qquad (1)$$

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نستخدم القاعدة مرة ثانية لإيجاد:

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$$\int 2xe^{x+1}dx$$

$$u = 2x, \qquad\qquad dv = e^{x+1}dx$$

$$du = 2dx, \qquad\qquad v = e^{x+1}$$

$$\int 2xe^{x+1}dx = 2xe^{x+1} - \int 2e^{x+1}dx = 2xe^{x+1} - 2e^{x+1} + C \qquad (2)$$

من **(1) , (2)** نحصل على:

$$\int x^2 e^{x+1}\, dx = x^2 e^{x+1} - 2xe^{x+1} + 2e^{x+1} + C$$

$$= (x^2 - 2x + 2)e^{x+1} + C$$

(14)

$$\int x^2 e^{2x-3}\, dx$$

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$$u = x^2, \qquad\qquad dv = e^{2x-3}dx$$

$$du = 2xdx, \qquad\qquad v = \frac{1}{2}e^{2x-3}$$

$$\int u\, dv = uv - \int v\, du$$

$$\int x^2 e^{2x-3}\, dx = \frac{1}{2}x^2 e^{2x-3} - \int xe^{2x-3}\, dx \qquad\qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int xe^{2x-3}dx$$

$$u = x, \qquad\qquad dv = e^{2x-3}dx$$

$$du = dx, \qquad\qquad v = \frac{1}{2}e^{2x-3}$$

$$\int xe^{2x-3}\,dx = \frac{1}{2}xe^{2x-3} - \int \frac{1}{2}e^{2x-3}\,dx = \frac{1}{2}xe^{2x-3} - \frac{1}{4}e^{2x-3} + C \qquad (2)$$

من (2) , (1) نحصل على:

$$\int x^2 e^{2x-3}\,dx = \frac{1}{2}x^2 e^{2x-3} - \frac{1}{2}xe^{2x-3} + \frac{1}{4}e^{2x-3} + C$$

$$= \left(\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}\right)e^{2x-3} + C$$

(15)

$$\int (\ln x)^2\,dx$$

$$u = (\ln x)^2, \qquad\qquad dv = dx$$

$$du = 2(\ln x)\frac{dx}{x}, \qquad\qquad v = x$$

$$\int u\,dv = uv - \int v\,du$$

$$\int (\ln x)^2\,dx = x\,(\ln x)^2 - \int 2\ln x\,dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int 2\ln x\,dx$$

$$u = 2\ln x, \qquad\qquad dv = dx$$

$$du = \frac{2}{x}dx, \qquad\qquad v = x$$

$$\int 2\ln x\,dx = 2x\ln x - \int 2\,dx = 2x\ln x - 2x + C \qquad (2)$$

من (2) , (1) نحصل على:

$$\int (\ln x)^2\,dx = x(\ln x)^2 - 2x\ln x + 2x + C$$

(16)

$$\int e^{2x} \sin x \, dx$$

$$u = e^{2x}, \qquad\qquad dv = \sin x \, dx$$

$$du = 2e^{2x} dx, \qquad\qquad v = -\cos x$$

$$\int u \, dv = uv - \int v \, du$$

$$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2 \int e^{2x} \cos x \, dx \qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int e^{2x} \cos x \, dx$$

$$u = e^{2x}, \qquad\qquad dv = \cos x \, dx$$

$$du = 2e^{2x} dx, \qquad\qquad v = \sin x$$

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$$\int e^{2x} \cos x \, dx = e^{2x} \sin x - 2 \int e^{2x} \sin x \, dx \qquad (2)$$

نعوض (2) في (1) :

$$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2 \left(e^{2x} \sin x - 2 \int e^{2x} \sin x \, dx \right)$$

$$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x - 4 \int e^{2x} \sin x \, dx$$

$$5 \int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x$$

$$\therefore \int e^{2x} \sin x \, dx = \frac{1}{5} (-e^{2x} \cos x + 2e^{2x} \sin x)$$

(17)

$$\int \sin(\ln x)\, dx$$

$$u = \sin(\ln x), \qquad\qquad dv = dx$$

$$du = \frac{\cos(\ln x)}{x}\, dx, \qquad\qquad v = x$$

$$\int u\, dv = uv - \int v\, du$$

$$\int \sin(\ln x)\, dx = x\sin(\ln x) - \int \cos(\ln x)\, dx \qquad\qquad (1)$$

نستخدم القاعدة مرة ثانية لإيجاد:

$$\int \cos(\ln x)\, dx$$

$$u = \cos(\ln x), \qquad\qquad dv = dx$$

$$du = \frac{-\sin(\ln x)}{x}\, dx, \qquad\qquad v = x$$

$$\int \cos(\ln x)\, dx = x\cos(\ln x) + \int \sin(\ln x)\, dx \qquad\qquad (2)$$

نعوض (2) في (1) :

$$\int \sin(\ln x)\, dx = x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\, dx$$

$$2\int \sin(\ln x)\, dx = x\sin(\ln x) - x\cos(\ln x) + C$$

$$\therefore \int \sin(\ln x)\, dx = \frac{1}{2}x(\sin(\ln x) - \cos(\ln x)) + C$$

1.   **a**

2.   **a**

3.   **a**

4.   **b**

5.   **a**

6.   **b**

7.   **b**

8.   **d**

9.   **b**

10.   **b**

11.   **c**