نماذج أجابة أسئله أمتحان تقييمى أول

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(1)

مثال: أوجد التكامل التالي:

$$\int \left(\frac{3x^2 - x}{x}\right)^2 dx$$

الحل:

$$\int \left(\frac{3x^2 - x}{x}\right)^2 dx = \int \left(\frac{3x^2}{x} - \frac{x}{x}\right)^2 dx = \int (3x-1)^2 dx$$

$$= \int (3x-1)^2 dx = \int (9x^2 - 6x + 1) dx$$

$$= \frac{9x^3}{3} - \frac{6x^2}{2} + x + C = 3x^3 - 3x^2 + x + C$$

(2)

$$\int (x+2)\sqrt[3]{x^2+4x-1}\,dx$$

الحل :

بفرض $u = x^2 + 4x - 1$

$du = (2x+4)dx \quad , \quad \frac{1}{2}du = (x+2)dx$

$$\int (x+2)\sqrt[3]{x^2+4x-1}\,dx = \int u^{\frac{1}{3}}\left(\frac{1}{2}\,du\right)$$

$$= \frac{1}{2}\int u^{\frac{1}{3}}\,du$$

$$= \frac{1}{2}\left[\frac{3}{4}\,u^{\frac{4}{3}}\right] + C$$

$$\therefore \int (x+2)\sqrt[3]{x^2+4x-1}\,dx = \frac{3}{8}(\,x^2+4x-1\,)^{\frac{4}{3}} + C$$

(3)

$$\int \sqrt[3]{x^2 - 5x + 2}\ \ (2x - 5)dx$$

الحل:

$$\int \sqrt[3]{x^2 - 5x + 2}\ \ (2x - 5)dx = \int (x^2 - 5x + 2)^{\frac{1}{3}}\ (2x - 5)dx$$

$$u = x^2 - 5x + 2 \Longrightarrow du = (2x - 5)dx$$

$$\int \sqrt[3]{x^2 - 5x + 2}\ \ (2x - 5)dx = \int u^{\frac{1}{3}}\, du = \frac{u^{\frac{4}{3}}}{\frac{4}{3}} + C$$

$$= \frac{3}{4}(x^2 - 5x + 2)^{\frac{4}{3}} + C$$

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(4)

$$\int x(2x - 1)^3\, dx$$

الحل:

$$u = 2x - 1 \Longrightarrow du = 2dx \Longrightarrow \frac{du}{2} = dx$$

$$u = 2x - 1 \Longrightarrow 2x = u + 1 \Longrightarrow x = \frac{u + 1}{2}$$

$$\int x(2x - 1)^3\, dx = \int \left(\frac{u + 1}{2}\right) u^3 \frac{du}{2} = \frac{1}{4}\int (u^4 + u^3)du$$

$$= \frac{1}{4}\left(\frac{u^5}{5} + \frac{u^4}{4} + C\right) = \frac{1}{20}(2x - 1)^5 + \frac{1}{16}(2x - 1)^4 + C$$

(5)

$$\int \csc^5 x \ \cot x \, dx$$

الحل:

$$u = \csc x \Rightarrow du = -\csc x \cot x \, dx \Rightarrow -du = \csc x \cot x \, dx$$

$$\int csc^5 x . \cot x \, dx = \int \csc^4 x . \csc x . \cot x \, dx = \int u^4 (-du)$$

$$= -\int u^4 \, du = -\frac{u^5}{5} + C = -\frac{1}{5}\csc^5 x + C$$

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(6)

$$\int \frac{x-1}{\sqrt{x}+1} dx$$

$$\int \frac{(\sqrt{x}-1)\cancel{(\sqrt{x}+1)}}{\cancel{(\sqrt{x}+1)}} dx$$

$$= \int (\sqrt{x}-1) \, dx$$

$$= \int (x^{\frac{1}{2}}-1) \, dx$$

$$= \frac{2}{3} x^{\frac{3}{2}} - x + C$$

$$= \frac{2}{3}\sqrt{x^3} - x + C$$

(7)

أوجد: $\int \frac{x+1}{\sqrt[3]{x+1}}\,dx$

الحل

$= \int \frac{(\sqrt[3]{x}+1)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}{(\sqrt[3]{x}+1)}\,dx$

تذكر أن: تحليل

$(x+1) = (\sqrt[3]{x}+1)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)$

$= \int (\sqrt[3]{x^2}-\sqrt[3]{x}+1)\,dx$

$= \int (x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1)\,dx$

$= \frac{3}{5}x^{\frac{5}{3}} - \frac{3}{4}x^{\frac{4}{3}} + x + c$

$= \frac{3}{5}\sqrt[3]{x^5} - \frac{3}{4}\sqrt[3]{x^4} + x + c$

(8)

إذا كان: $F(x) = \int (2x+5)dx$ ، $F(-1) = 0$ فأوجد $F(x)$

الحل

$F(x) = \int (2x+5)\,dx$

$F(x) = 2 \cdot \frac{1}{2}x^2 + 5x + c$

$F(x) = x^2 + 5x + c$

$\because F(-1) = 0$

$(-1)^2 + 5(-1) + c = 0$ shift-solve

$c = 4$

$\therefore F(x) = x^2 + 5x + 4$

(9)

الحل

$$I = 3\int \frac{(x^{\frac{1}{3}}-5)}{x^{\frac{2}{3}}}\,dx$$

$$= 3\int (x^{\frac{1}{3}}-5)\cdot \underline{x^{-\frac{2}{3}}\,dx} \leftarrow \text{اعادة صياغة}$$

$u = \underline{x^{\frac{1}{3}}-5}$

$du = \frac{1}{3}x^{-\frac{2}{3}}\,dx$

$3\,du = \underline{x^{-\frac{2}{3}}\,dx}$

مضروب

$$= 3\int u\cdot 3du$$

$$= 9\int u\,du$$

$$= 9\cdot\frac{1}{2}u^2 + C$$

$$= \frac{9}{2}(x^{\frac{1}{3}}-5)^2 + C$$

$$= \frac{9}{2}(\sqrt[3]{x}-5)^2 + C$$

(10)

أوجد: $\int x(3x+2)^6 dx$

الحل

$I = \int (3x+2)^6 \cdot \boxed{x}\, dx$ ← إعادة صياغة

$= \int u^6 \cdot \frac{1}{3}(u-2) \cdot \frac{1}{3} du$

ضرب

$= \frac{1}{9} \int u^6 (u-2)\, du$

$= \frac{1}{9} \int (u^7 - 2u^6)\, du$

$= \frac{1}{9} \left[\frac{1}{8} u^8 - 2 \cdot \frac{1}{7} u^7 \right] + c$

$= \frac{1}{72} u^8 - \frac{2}{63} u^7 + c$

$= \frac{1}{72}(3x+2)^8 - \frac{2}{63}(3x+2)^7 + c$

إبدأ

$u = 3x+2$

$du = 3dx$

$\frac{1}{3} du = dx$

$u = 3x+2$

$3x+2 = u$

$3x = u-2$

$\frac{3x}{3} = \frac{u-2}{3}$

$\boxed{x} = \frac{1}{3}(u-2)$

(11)

أوجد: $\int (x^2-2)e^{x^3-6x}\,dx$

الحل

$I = \int e^{x^3-6x} \cdot \underline{(x^2-2)\,dx}$

$= \int e^u \cdot \frac{1}{3}\,du$

$= \frac{1}{3}e^u + c$

$= \frac{1}{3}e^{x^3-6x} + c$

$u = x^3 - 6x$

$du = (3x^2-6)\,dx$

$du = 3(x^2-2)\,dx$

$\frac{1}{3}du = \underline{(x^2-2)\,dx}$

(12)

أوجد مشتقة كل من الدوال التالية:

a $f(x) = e^{\frac{2x}{3}}$

الحل

$f'(x) = e^{\frac{2x}{3}} \cdot \ln e \cdot \left(\frac{2x}{3}\right)' = \frac{2}{3}e^{\frac{2}{3}x}$

$\boxed{\ln e = 1}$

b $g(x) = e^{x^2+3x-1}$

الحل

$g'(x) = (x^2+3x-1)' \cdot e^{x^2+3x-1} = (2x+3) \cdot e^{x^2+3x-1}$

c $h(x) = e^{\sec x}$

الحل

$h'(x) = (\sec x)' \cdot e^{\sec x} = \sec x \tan x \cdot e^{\sec x}$

(13)

أوجد مشتقات كلّ من الدوال التالية:

$f(x) = \ln x^2$

الحل

$f'(x) = \frac{2}{x}$

$h(x) = \ln \sqrt{x}$

الحل

$h(x) = \ln x^{\frac{1}{2}}$

$h'(x) = \frac{1}{2x}$

$g(x) = \ln\left(\frac{1}{x}\right)$

الحل

$g(x) = \ln x^{-1}$

$g'(x) = \frac{-1}{x}$

$g(x) = \ln \frac{1}{2x+1}$

الحل

$g(x) = \ln(2x+1)^{-1} = -\ln(2x+1)$

$g'(x) = -\frac{2}{2x+1}$

(14)

$$y = ln(ln\, x)$$

$$\frac{dy}{dx} = \frac{1}{ln\, x} \cdot (ln\, x)'$$

$$= \frac{1}{ln\, x} \cdot \left(\frac{1}{x}\right)$$

$$= \frac{1}{x\, ln\, x}$$

....

$$y = ln\left(\frac{1}{x^2}\right)$$

$$\frac{dy}{dx} = \frac{1}{\frac{1}{x^2}} \cdot \left(\frac{1}{x^2}\right)'$$

$$= x^2 \cdot \frac{-2x}{x^4}$$

$$= \frac{-2}{x}$$

(15)

$$\int \frac{e^x}{e^x+1}dx$$

الحل

$u = e^x + 1$

$du = e^x \, dx$

$I = \int \frac{1}{u} du$

$= \ln|u| + c$

$= \ln|e^x + 1| + c$

(16)

$$\int \left(e^{0 \cdot 5x} + \frac{0 \cdot 5}{x} \right) dx$$

$$= \frac{1}{0 \cdot 5} e^{0 \cdot 5x} + 0 \cdot 5 \ln|x| + c$$

(17)

$$\int \left(\frac{x^3 - x}{x^4 - 2x^2}\right) dx$$

$$u = x^4 - 2x^2$$

$$du = 4x^3 - 4x$$

$$\int \frac{x^3 - x}{x^4 - 2x^2}\, dx =$$

$$\frac{1}{4}\int \frac{4(x^3 - x)}{x^4 - 2x^2}\, dx = \frac{1}{4}\int \frac{du}{u}$$

$$= \frac{1}{4}\ln|u| + c$$

$$= \frac{1}{4}\ln|x^4 - 2x^2| + c$$

(18)

$$\int \left(\frac{2}{3x+1}\right) dx$$

$$u = 3x + 1$$

$$du = 3$$

$$= \int \frac{2}{3x+1}\ dx$$

$$= \frac{2}{3}\int \frac{3}{3x+1} dx$$

$$= \frac{2}{3}\int \frac{du}{u} dx$$

$$= \frac{2}{3}(\ ln|u| + c)$$

$$= \frac{2}{3}\ ln|3x+1| + c$$